Higgsless GUT Breaking and Trinification

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Abstract

Boundary conditions on an extra-dimensional interval can be chosen to break bulk gauge symmetries and to reduce the rank of the gauge group. We consider this mechanism in models with gauge trinification. We determine the boundary conditions necessary to break the trinified gauge group directly down to that of the standard model. Working in an effective theory for the gauge symmetry-breaking parameters on a boundary, we examine the limit in which the GUT-breaking sector is Higgsless and show how one may obtain the low-energy particle content of the minimal supersymmetric standard model. We find that gauge unification is preserved in this scenario, and that the differential gauge coupling running is logarithmic above the scale of compactification. We compare the phenomenology of our model to that of four-dimensional trinified theories.

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I. INTRODUCTION

Extra spatial dimensions allow for the possibility of gauge symmetry breaking by the appropriate choice of boundary conditions on the fields. The relevance of this point to model building was first realized by Kawamura [1], in the context of SU(5) grand unified theories (GUTS), and was developed substantially afterwards by a number of authors [2]. In the simplest case of an S^1/Z_2 orbifold, the matrix representing the action of the Z_2 symmetry in field space may not commute with all the generators of the gauge symmetry. Boundary conditions may be chosen so that different components of the gauge multiplet have different Z_2 parities, leaving only some with zero modes after the theory is dimensionally reduced. The fact that the zero-mode spectrum includes incomplete multiplets of the gauge group indicates that the symmetry has been broken. Although no Higgs fields are involved, longitudinal gauge boson scattering amplitudes are well behaved at high energies [3]. The same approach may be employed to project away the zero-modes [4] of the color-triplet Higgs in SU(5) GUTS, naturally resolving the doublet-triplet splitting problem [1, 2].

In the simplest orbifold constructions, the orbifold parity commutes with the diagonal generators of the original gauge symmetry, so that the unbroken subgroup has the same rank. For symmetry breakings like SU(5)→SU(3)$_C$×SU(2)$_W$×U(1)$_Y$, [1, 2] or SU(3)$_W$ →SU(2)$_W$×U(1)$_Y$ [5, 6], the breaking by orbifold boundary conditions provides an economical approach for constructing models. However, larger groups, like E_6 or E_8 can only be broken directly to the standard model gauge group and, at best, a product of additional U(1) factors [7]. One must then rely on the conventional Higgs mechanism to complete the breaking of the residual GUT symmetry. In this paper, we will consider the use of more general boundary conditions to break such unified symmetries directly to the standard model gauge group, and hence, to reduce the rank of the original group. This approach has been discussed in the context of Higgsless electroweak symmetry breaking [3, 8]; here we will employ the same technique at a high scale, while retaining the ordinary Higgs mechanism for the breaking of electroweak symmetry. This choice allows us to eliminate the often complicated and problematic GUT-breaking Higgs sector, while allowing for the easy generation of light fermion masses.

The   unified   theory   we   consider   is   based   on   the   'trinified'   gauge   group

$G_T = \text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R \ltimes Z_3$ [9, 10, 11, 12, 13, 14]. The semidirect product (indicated by the symbol $\ltimes$) provides for a symmetry that cyclically permutes the gauge group labels C, L, and R. Hence, the $\text{SU}(3)^3$ representation (rep) $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ is part of the trinified rep

$$\mathbf{27} = (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \ . \tag{1.1}$$

Moreover, the Z_3 symmetry assures the equality of the three $\text{SU}(3)$ gauge couplings at the GUT scale. As originally pointed out in Ref. [9], an appropriate embedding of $\text{U}(1)_Y$ in $\text{SU}(3)_L \times \text{SU}(3)_R$ yields the familiar GUT-scale prediction $\sin^2 \theta = 3/8$. We review this construction in Section II. We will work with a supersymmetric trinified theory in which the G_T gauge multiplet may propagate in a single extra dimensional interval. We first consider the simplest case in which all the matter and Higgs fields are confined to a brane on which G_T is broken. Working in an effective theory of gauge-symmetry-breaking 'spurions' on this brane, we establish the boundary conditions necessary to break the bulk gauge group to that of the standard model, $G_T \rightarrow G_{SM}$. We also include the couplings of these spurions to the matter multiplets of the theory. In the limit in which the symmetry breaking parameters are taken to infinity, we obtain the Higgsless limit of the GUT-breaking sector. In particular, the mass scale for the heavy gauge multiplets becomes determined by the compactification radius, and all exotic matter fields are decoupled from the theory. The low-energy theory is simply that of the minimal supersymmetric standard model (MSSM), with a set of massive gauge multiplets at a scale lower than that of conventional supersymmetric unification, 2×10^{16} GeV. We show that unification is nonetheless preserved. Above the compactification scale, the differential gauge running (i.e., $\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu)$ for $i \neq j$) is logarithmic, a feature that has been noted before in the case of $\text{SU}(5)$ GUTS broken on a boundary [15]. We then show that viable alternative theories exist in which the Higgs and/or matter multiplets are allowed to propagate in the bulk space, and we discuss the boundary conditions on these fields. In this case, the exotic matter fields remain part of the theory, but with large masses set by the compactification radius.

Our paper is organized as follows. In Section II, we review the symmetry breaking in conventional trinification models, and describe some of the main phenomenological features of these theories. In Section III, we give the extra-dimensional construction of supersymmetric $\text{SU}(3)^3$, determine the boundary conditions necessary to break the gauge group down

to that of the standard model, and study the Higgsless limit of the GUT-breaking sector. In Section IV, we study gauge unification in our minimal model, while in Section V we discuss the possibility of allowing chiral multiplets in the bulk. In Section VI, we summarize our conclusions.

II.   FRAMEWORK

Trinification [9, 10, 11, 12, 13, 14] is based on the gauge group G_T =SU$(3)_C \times$SU$(3)_L \times$SU$(3)_R \ltimes Z_3$, where $\ltimes$ indicates a semidirect product. The Z_3 symmetry cyclically permutes the gauge group labels C, L and R, ensuring a single unified coupling at the GUT scale. G_T reps consist of the sum of cyclically permuted SU$(3)^3$ reps. For example, the gauge fields are in the **24**-dimensional rep

$$A_T^\mu(\mathbf{24}) = A_C^\mu(\mathbf{8},\mathbf{1},\mathbf{1}) + A_L^\mu(\mathbf{1},\mathbf{8},\mathbf{1}) + A_R^\mu(\mathbf{1},\mathbf{1},\mathbf{8}). \tag{2.1}$$

Here, A_C^μ represent the eight gluon fields of the standard model, while only some of the A_L^μ and A_R^μ above correspond to electroweak gauge bosons. The SU$(2)_W$ gauge group of the standard model is contained entirely in SU$(3)_L$; writing $A = A^a T^a$, then the SU$(2)_W$ gauge bosons W^a correspond to A_L^a for $a = 1 \ldots 3$. On the other hand, the hypercharge gauge boson is a linear combination of A_L^8, A_R^3 and A_R^8. The choice

$$A_Y^\mu = -\frac{1}{\sqrt{5}}(A_L^8 + \sqrt{3}A_R^3 + A_R^8)^\mu, \tag{2.2}$$

yields the standard GUT-scale prediction $\sin^2 \theta_W = 3/8$. The pattern of gauge symmetry breaking is achieved via one or more Higgs fields in the **27**-dimensional rep,

$$\phi(\mathbf{27}) = \phi(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) + \phi(\mathbf{3},\bar{\mathbf{3}},\mathbf{1}) + \phi(\bar{\mathbf{3}},\mathbf{1},\mathbf{3}) \ . \tag{2.3}$$

Only the first SU$(3)^3$ factor in this rep allows for color-singlet vacuum expectation values (vevs) that may break $SU(3)^3$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\phi(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) = \begin{pmatrix} \hat{0} & 0 & 0 \\ 0 & \hat{0} & \hat{0} \\ 0 & v_2 & v_1 \end{pmatrix} \ . \tag{2.4}$$

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Here, v_i represent the GUT-scale vevs, while hatted entries denote components capable of eventually breaking the electroweak gauge group. Spontaneous symmetry breaking renders twelve of the original gauge bosons with masses of order the GUT scale. Interestingly, these massive gauge bosons are integrally charged and cannot generate dimension-six operators that contribute to proton decay. Depending on the number of Higgs multiplets and their couplings to the matter fields, proton decay may still occur via color-triplet Higgs exchange.

Standard model fermions are embedded economically in the **27**-dimensional representation. In SU(5) language, the **27** decomposes as

$$\mathbf{27} = [\mathbf{10} \oplus \bar{\mathbf{5}}] \oplus \mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{1} \oplus \mathbf{1} \ . \tag{2.5}$$

The reps in brackets correspond to a full standard model generation, while the remaining reps are exotic. Thus the exotic fields include left- and right-handed fermions with the quantum numbers of a charge $-1/3$ weak singlet quark (B), a hypercharge $-1/2$ weak doublet lepton (E^0, E^-) and an electroweak singlet (N). Using the notation

$$\psi(\mathbf{27}) \ = \ \psi(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}) + \psi(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}) + \psi(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \tag{2.6}$$

$$\equiv \ \psi_C + \psi_L + \psi_R \ , \tag{2.7}$$

we may choose an SU(2)$_W$ basis in which the fermion reps take the matrix form

$$\psi_c \ = \ \begin{pmatrix} E^{0c} & E & e \\ -E^c & E^0 & \nu \\ e^c & N^c & N \end{pmatrix}, \ \psi_L = \begin{pmatrix} u_{\bar{r}}^c & u_{\bar{g}}^c & u_{\bar{b}}^c \\ d_{\bar{r}}^c & d_{\bar{g}}^c & d_{\bar{b}}^c \\ B_{\bar{r}}^c & B_{\bar{g}}^c & B_{\bar{b}}^c \end{pmatrix}, \ \psi_R = \begin{pmatrix} u_r & d_r & B_r \\ u_g & d_g & B_g \\ u_b & d_b & B_b \end{pmatrix}, \tag{2.8}$$

where all entries are left-handed. In supersymmetric trinification, these matrices are composed of left-handed chiral superfields, with each entry indicating the fermionic component. Yukawa couplings necessarily involve invariants formed by taking the product of three **27**'s. These come in two types,

$$Z_3[\psi_R \psi_L \phi_{Ci}] \ , \tag{2.9}$$

and

$$Z_3[\psi_C \psi_C \phi_{Cj}] \ . \tag{2.10}$$

We use the symbol Z_3 to represent the cyclic permutation of R, L and C, *e.g*,

$$Z_3[\psi_R \psi_L \phi_C] = \psi_R \psi_L \phi_C + \psi_C \psi_R \phi_L + \psi_L \psi_C \phi_R. \tag{2.11}$$

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The index on the field ϕ_C takes into account the possibility that there may be more than one **27**-plet Higgs field. If there is only one Higgs **27**, then both the up- and down-type quark Yukawa couplings for a given generation originate from a single G_T-invariant interaction, of the form shown in Eq. (2.9). This implies the incorrect GUT-scale mass relation [9]

$$\frac{m_u}{m_d} = \frac{m_c}{m_s} = \frac{m_t}{m_b}. \tag{2.12}$$

Therefore, at least two Higgs **27**'s must couple to the quarks via Eq. (2.9). Generally, the same set of Higgs fields will couple to the leptons via Eq. (2.10) and proton decay may proceed via color-triplet Higgs exchange. If a third Higgs **27**-plet is introduced that couples to the leptons only, then proton decay can be prevented by imposing a global symmetry on the Higgs sector that prevents mixing between the third Higgs and the other two. This, however, leads to a symmetry-breaking sector that seems somewhat contrived.

It is conventionally assumed that the vevs v_1 and v_2 arise in separate Higgs **27**-plets:

$$\phi(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v_1 \end{pmatrix}, \quad \chi(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & v_2 & 0 \end{pmatrix}. \tag{2.13}$$

The superpotential terms responsible for quark and lepton masses can now be determined from the invariants Eq.(2.9) and Eq.(2.10),

$$W_Q = (\psi_L)_i^j (\psi_R)_k^i [g_1(\phi_C)_j^k + g_2(\chi_C)_j^k] \ , \tag{2.14}$$

$$W_L = \frac{1}{2} h(\psi_C)_\alpha^i (\psi_C)_\beta^j [h_1(\phi_C)_\gamma^k + h_2(\chi_C)_\gamma^k] \epsilon_{ijk} \epsilon^{\alpha\beta\gamma} \ . \tag{2.15}$$

These may be expanded, yielding

$$W = g_2 v_2 \, d^c B + g_1 v_1 B^c B + v_1 h_1 \, \epsilon_{ij} L_H^{ci} L_H^j - v_2 h_2 \epsilon_{ij} L_H^{ci} L^j \ , \tag{2.16}$$

where the lepton doublets are defined by $L_H = (E^0, E)$, $L = (\nu, e)$, and $L_H^c = (-E^c, E^{c0})$. Clearly, one linear combination of B^c and d^c, and of L_H and L, remain unaffected by GUT symmetry breaking[1], and should be identified with the physical right-handed down quark

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[1] Ref. [10] states that no light lepton eigenstate will remain if $h_2 \neq 0$. This is not correct, since unbroken electroweak symmetry assures that a massless eigenstate must remain.

and lepton doublet superfields:

$$d^c_{phys} = (-g_2 v_2 \, B^c + g_1 v_1 \, d^c)/\sqrt{g_1^2 v_1^2 + g_2^2 v_2^2}$$
$$L_{phys} = (h_2 v_2 \, L_H + h_1 v_1 \, L)/\sqrt{h_1^2 v_1^2 + h_2^2 v_2^2} \ . \tag{2.17}$$

The masses of the heavy quark and lepton states remaining in Eq. (2.16) are given by

$$m_{B,B^c_{phys}} = (g_1^2 v_1^2 + g_2^2 v_2^2)^{1/2}, \tag{2.18}$$

$$m_{L^c_H, L_{H,phys}} = (h_1^2 v_1^2 + h_2^2 v_2^2)^{1/2} \ . \tag{2.19}$$

For this minimal choice of symmetry breaking, the singlets N^c and N remain massless. However, as we discuss in the next section, vevs in other Higgs field representations can give masses to these states as well.

We will not discuss the structure of the Higgs sector in conventional trinified theories since our goal is to dispense with this sector entirely. We henceforth consider supersymmetric trinified theories embedded in $4 + 1$ spacetime dimensions. As in Ref. [3], we assume that the extra spatial dimension is compact, and runs over the interval $y = 0$ to $y = \pi R$. We will always assume that the G_T gauge multiplet propagates in the bulk, and we will consider consistent boundary conditions that allow us to break this gauge group directly to that of the standard model upon compactification. The radius of compactification is a free parameter that we will determine based on the condition that supersymmetric gauge unification is preserved. We first consider the simplest case in which all matter and Higgs fields are placed on the $y = \pi R$ brane, and afterwards discuss the possibility of placing chiral multiplets in the bulk.

In all cases, we will treat the symmetry breaking on the πR brane in an effective theory approach. We will introduce G_T breaking spurions $\{\Phi_i\}$ on this brane and consider both their couplings to brane-localized fields, as well as their effect on the 5D wave function of fields in the bulk. Historically, the term "spurion" refers to a symmetry-breaking parameter that is taken to transform as a spurious field, so that it may be included consistently in an effective Lagrangian. In the present case, one may think of the spurions as a collection of brane Higgs vevs, that can plausibly arise in some ultraviolet completion. Since we will focus on the limit in which these vevs are taken to infinity, we will not defend any particular ultraviolet theory. Partial examples will be given only to justify the consistency of the

boundary conditions that we assume. In a few instances, we will require higher-dimension operators involving the spurions, which necessarily involve some cut off Λ. In the decoupling limit, we will take both Φ and Λ to infinity in fixed ratio. In other words, we do not assign Λ to some physical scale, but use this limiting procedure to obtain a consistent Higgsless low-energy effective theory that could otherwise be defined *ab initio*.

III. SYMMETRY BREAKING

We choose to break the trinified gauge group at the $y = \pi R$ brane. For a generic gauge field A^μ, the boundary conditions

$$\partial_5 A^\mu(x^\nu, 0) = 0 \quad \text{and} \quad \partial_5 A^\mu(x^\nu, \pi R) = V A^\mu(x, \pi R) \tag{3.1}$$

lead to a mode expansion of the form

$$f_k(y) = N_k \cos(M_k y) \ , \tag{3.2}$$

where M_k is given by the transcendental equation

$$M_k \tan(M_k \pi R) = -V \ , \tag{3.3}$$

and where the normalization

$$N_k = \frac{\sqrt{2}}{\sin(M_k \pi R)} [\pi R(1 + M_k^2/V^2) - 1/V]^{-1/2} \tag{3.4}$$

assures that $\int_0^{\pi R} f^2 = 1$ [3]. Note that the symmetry breaking parameter V has dimensions of mass. The nontrivial boundary condition in Eq. (3.1) can be realized in an ultraviolet completion of the theory in which a brane localized Higgs field σ is responsible for the symmetry breaking. The brane equations of motion for the field A^μ includes terms localized at $y = \pi R$ from the start, as well as surface terms obtained from integrating the bulk action by parts. In particular, the kinetic terms

$$S_{KE} \supset \int d^4x \int_0^{\pi R} dy \, [-\frac{1}{2} F_{5\nu} F^{5\nu} + D^\mu \sigma^\dagger D_\mu \sigma \, \delta(y - \pi R)] \tag{3.5}$$

include

$$S_{KE} \supset \int d^4x \int_0^{\pi R} dy \, [-\partial_5 A_\nu \partial_5 A^\nu + \frac{g_5^2}{2} \langle\sigma\rangle^\dagger \langle\sigma\rangle A^\mu A_\mu \, \delta(y - \pi R)] \ . \tag{3.6}$$

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Variation of this portion of the action with respect to A^ν yields a constraint at $y = \pi R$,

$$-\partial_5 A_\nu + \frac{g_5^2}{2}\langle\sigma\rangle^\dagger\langle\sigma\rangle A^\nu = 0 \ , \qquad (3.7)$$

which corresponds to the desired boundary condition if one identifies $V \equiv g_5^2\langle\sigma\rangle^\dagger\langle\sigma\rangle/2$. Since the 5D gauge coupling g_5 has mass dimension $-1/2$, one finds that V has dimensions of mass, as before.

Csáki, Grojean, Murayama, Pilo, and Terning [3], have demonstrated that the boundary conditions given in Eq. (3.1) require a brane-localized Higgs field to cancel contributions to scattering amplitudes that grow with energy as E^2. However, a remarkable feature of brane-localized breaking of gauge symmetries is that one can decouple the Higgs field without decoupling the massive gauge multiplets as well. In the limit that $\langle\sigma\rangle$, and hence V, are taken to infinity, one finds from Eq. (3.3) that the KK mass spectrum becomes

$$M_n \approx \frac{M_c}{2}(2n+1)(1 + \frac{M_c}{\pi V} + \cdots) \ , \qquad (3.8)$$

where M_c is the compactification scale $1/R$. Thus, the low-energy theory has no Higgs fields, and the KK tower for the gauge fields is shifted by $+M_c/2$ relative to the tower one would obtain if V were set to zero.

In the case of G_T, the first SU(3) factor corresponds to the unbroken color group, so we may immediately write down the boundary conditions on the gluon fields A_C^μ,

$$\partial_5 A_C^\mu(x,0) = \partial_5 A_C^\mu(x,\pi R) = 0 \ . \qquad (3.9)$$

Similarly, an SU(2) subgroup of the second SU(3) factor remains unbroken, so that

$$\partial_5 A_L^a(x,0) = \partial_5 A_L^a(x,\pi R) = 0 \text{ for } a = 1\ldots3 \qquad (3.10)$$

Since the only remaining unbroken group is a U(1) factor, all gauge fields corresponding to off-diagonal generators must become massive. Thus, we require that

$$\partial_5 A_L^a(x,0) = 0 \ , \ \partial_5 A_L^a(x,\pi R) = V_L A_L^a(x,\pi R) \text{ for } a = 4\ldots7 \ , \qquad (3.11)$$

$$\partial_5 A_R^a(x,0) = 0 \ , \ \partial_5 A_R^a(x,\pi R) = V_R A_R^a(x,\pi R) \text{ for } a = 1,2,4\ldots7 \ . \qquad (3.12)$$

The remaining U(1) factors are more interesting. As we showed in the previous section, the embedding of hypercharge within SU(3)$_L\times$SU(3)$_R$ that leads to the prediction $\sin^2\theta = 3/8$

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requires that the hypercharge gauge boson be identified with the linear combination

$$A_Y^\mu = -\frac{1}{\sqrt{5}}(A_L^8 + \sqrt{3}A_R^3 + A_R^8)^\mu \tag{3.13}$$

Thinking in terms of an ultraviolet completion, suitable Higgs fields must generate a brane gauge boson mass matrix with a zero eigenvalue corresponding to the eigenvector $(-1/\sqrt{5}, -\sqrt{3}/\sqrt{5}, -1/\sqrt{5})$. The only other necessary constraint on this matrix is that the remaining eigenvalues must be non-vanishing. Restricting ourselves to real entries, for the sake of simplicity, we may parameterize the remaining boundary conditions as follows:

$$\partial_5 A_L^8(x,0) = \partial_5 A_R^3(x,0) = \partial_5 A_R^8(x,0) = 0 \ , \tag{3.14}$$

$$\partial_5 \begin{bmatrix} A_L^8(x,\pi R) \\ A_R^3(x,\pi R) \\ A_R^8(x,\pi R) \end{bmatrix} = \begin{pmatrix} V_1 & -\frac{1}{2\sqrt{3}}(V_1 + V_3) & -\frac{1}{2}(V_1 - V_3) \\ -\frac{1}{2\sqrt{3}}(V_1 + V_3) & \frac{1}{6}(V_2 + V_3) & \frac{1}{2\sqrt{3}}(V_1 - V_2) \\ -\frac{1}{2}(V_1 - V_3) & \frac{1}{2\sqrt{3}}(V_1 - V_2) & \frac{1}{2}(V_2 - V_3) \end{pmatrix} \begin{bmatrix} A_L^8(x,\pi R) \\ A_R^3(x,\pi R) \\ A_R^8(x,\pi R) \end{bmatrix} \tag{3.15}$$

Finally, we consider the A^5 components. In a nonsupersymmetric theory, we could impose the boundary conditions $A^5(x,0) = A^5(x,\pi R) = 0$ on all the gauge fields so that no additional light scalar states remain in the 4D theory. In the supersymmetric case, A^μ and A^5 live within a vector V and chiral Φ_V superfield, respectively. Since supersymmetry is unbroken, the fermionic components of V and Φ_V (say, λ and ψ) must form Dirac spinors with the same mass spectrum as the gauge fields [15]. Since these masses originate from terms of the form $\partial_5 \lambda \psi$, the 5D wave function of Φ_V must be proportional to $\sin M_k y$, with M_k given as before.

If one were to assume an ultraviolet completion involving only the minimal Higgs content of conventional 4D trinified theories (localized on the πR brane) one would find that

$$\begin{aligned} V_1 &= \frac{2}{3}(v_1^2 + v_2^2)g_5^2 \\ V_2 &= \frac{1}{3}(2\,v_1^2 + 5\,v_2^2)g_5^2 \\ V_3 &= -\frac{1}{3}(2\,v_1^2 - 4\,v_2^2)g_5^2 \ . \end{aligned} \tag{3.16}$$

The more general values of the parameters V_i may be thought of as arising in some arbitrarily complicated GUT-breaking Higgs sector, which decouples as one takes V_L, V_R and $V_i \to \infty$.

However, we will not wed ourselves to any particular interpretation of the physics responsible for generating the symmetry-breaking parameters on the boundary.

We will proceed with an effective field theory analysis of the possible symmetry breaking on the πR brane. We will introduce the symmetry breaking systematically in terms of constant spurion fields that we may treat as transforming in irreducible reps of $\mathrm{SU}(3)^3$. When we obtain operators that are nonrenormalizable, we will introduce powers of a cutoff, Λ to obtain the proper mass dimension, as discussed at the end of Section II.

A given spurion representation may contribute to the symmetry breaking parameterized by Eq. (3.15) provided that it contains standard model singlet components, with hypercharge defined as in Eq. (3.13), that develop vevs. We know immediately of one possibility from the minimal 4D trinified theory, namely a $\mathbf{27}$ with vevs in the $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ component,

$$
\Phi(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}) \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & v_2 & v_1 \end{pmatrix} . \tag{3.17}
$$

As described in Section II, these vevs give mass to the heavy fields B, B^c, L_H and L_H^c while contributing to the boundary condition on the gauge fields via Eq. (3.16). This rep, however, does not contribute to the mass of the new singlet leptons, N^c and N. Since we wish to retain only the particle content of the MSSM at the electroweak scale, we will be more general. The set of $\mathrm{SU}(3)^3$ representations that appear in the product of two $\mathbf{27}$'s and that are color singlet are $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$, $(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{3})$, $(\mathbf{1}, \mathbf{3}, \mathbf{6})$, and $(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$. For each, we may isolate the components that are $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ singlets. The results are shown in Table I. While the reps $(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{3})$ and $(\mathbf{1}, \mathbf{3}, \mathbf{6})$ contain standard model singlet components, it turns out that these do not split the $\mathbf{27}$ matter multiplets. For example, the coupling of the $(\mathbf{1}, \bar{\mathbf{6}}, \bar{\mathbf{3}})$ to two $\mathbf{27}$ matter superfields may be written

$$
W = \Psi_\alpha^a \Psi_\beta^b \Phi_{ab,\gamma}^{(\mathbf{1}, \bar{\mathbf{6}}, \bar{\mathbf{3}})} \epsilon^{\alpha\beta\gamma} \tag{3.18}
$$

which vanishes for $a = b = 3$ and $\gamma = 1$, because of the antisymmetry of the $\mathrm{SU}(3)_R$ epsilon tensor. Of the three new spurion reps in Table I, only the $(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$ gives us something new,

$$
\begin{aligned}
W &= \Psi_\alpha^a \Psi_\beta^b \Phi_{ab}^{\alpha\beta} \\
&= v_{22} N_R^2 + 2 v_{23} N_R N_L + v_{33} N_L^2 .
\end{aligned} \tag{3.19}
$$

TABLE I: SU(3)^3 reps in the product of two trinified **27**-plets containing Standard Model singlet components, with hypercharge defined as in Eq. (3.13). Parentheses delimit indices that are symmetric.

SU(3)^3 rep	SU(3)$_L \times$SU(3)$_R$ tensor	SM singlet components
$(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$	Φ^a_α	$a = 3, \alpha = 2, 3$
$(\mathbf{1}, \bar{\mathbf{6}}, \bar{\mathbf{3}})$	$\Phi_{(ab)\alpha}$	$a = b = 3, \alpha = 1$
$(\mathbf{1}, \mathbf{3}, \mathbf{6})$	$\Phi^{a(\alpha\beta)}$	$a = 3, (\alpha\beta) = (12), (13)$
$(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$	$\Phi^{(\alpha\beta)}_{(ab)}$	$(ab) = (33), (\alpha\beta) = (22), (23), (33)$

Here v_{ij} corresponds to vevs for the standard model singlet components of the $(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$ spurion, as given in Table I. Hence, we arrive at Majorana and Dirac masses for the exotic neutral leptons, which may be decoupled from the theory if the v_{ij} are taken to infinity. Thus we reach the following conclusion:

Gauge symmetry breaking spurions localized at the πR brane in the **27** and **108** irreducible reps of the trinification group, and with nonvanishing standard model singlet entries in their $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ and $(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$ components, respectively, break the trinification gauge group down to the standard model, and yield the MSSM matter content at low energies. In the limit that all the symmetry breaking parameters are taken to infinity, we obtain Higgsless trinification breaking with an incomplete matter multiplet located at the πR brane.

This picture is pleasing since any physics on the brane associated with an ultraviolet completion that might lead to proton decay has been decoupled away. The only issue we have not taken into account is the mechanism for breaking electroweak symmetry and the generation of light fermion masses. We may easily incorporate the standard Higgs mechanism for electroweak symmetry breaking by introducing **27** and $\overline{\mathbf{27}}$ Higgs superfields on the πR brane, Ψ_H and $\Psi_{\bar{H}}$, respectively. (These are distinguished from matter superfields by unbroken matter or R-parity, which we assume throughout.) We identify H ($\bar{H}$) as the doublet Higgs field with hypercharge 1/2 ($-1/2$) living inside the multiplet Ψ_H ($\Psi_{\bar{H}}$). We also introduce another spurion rep, the **192**, which includes the color singlet rep $\Omega \sim (\mathbf{1}, \mathbf{8}, \mathbf{8})$. Assuming that the nonvanishing, standard model singlet components of Ω are

given by

$$\Omega_{ab}^{\alpha\beta} = v_\Omega \, T^{8\,a}_{\quad b} \, T^{3\,\alpha}_{\quad \beta} \qquad (3.20)$$

then the couplings

$$W = H_\alpha^a (\mu \, \delta_\beta^\alpha \delta_a^b + h \, \Omega_{a\alpha}^{b\beta}) \bar{H}_b^\beta \qquad (3.21)$$

will provide high-scale μ terms for all members of the Higgs multiplet, except for the weak doublets H and $\bar{H}$, providing that $\mu = -4\sqrt{3} \, h \, v_\Omega$. Thus, in this approach, we simply impose a fine-tuning of the parameters to arrange for a doublet-triplet splitting [2]. However, since we ultimately take the limit in which $v_\Omega \to \infty$, as with the other symmetry-breaking spurions, there is no sign of this fine-tuning in the low-energy theory. From a low-energy perspective, it is completely consistent to assign two electroweak Higgs doublets to the brane in the GUT-Higgsless limit.

One feature of this solution that needs clarification is the coupling of these Higgs doublets to the matter fields. While the up-quark Higgs fields H lives in a $\mathbf{27}$ and couples to the matter fields via the conventional cubic interactions of 4D trinified theories, the down-type Higgs fields $\bar{H}$ lies in a $\overline{\mathbf{27}}$ and does not couple directly. Nonetheless, we may arrange for a suitable down quark Yukawa matrix by introducing a $\overline{\mathbf{27}}$ spurion with the same nonvanishing components as the $\mathbf{27}$ spurion that we have already considered. Then the down quark Yukawa matrix will originate via a higher-dimension operator

$$\frac{1}{\Lambda} Z_3 [\Phi(\mathbf{1},\overline{\mathbf{3}},\mathbf{3}) \bar{H}(\mathbf{1},\overline{\mathbf{3}},\mathbf{3}) \Psi(\overline{\mathbf{3}},\mathbf{1},\mathbf{3}) \Psi(\mathbf{3},\overline{\mathbf{3}},\mathbf{1})] \ . \qquad (3.22)$$

We may generate the down quark Yukawa couplings by fixing the ratio of the spurion vev to Λ, and taking both to infinity in the Higgsless limit.

IV. GAUGE UNIFICATION

By breaking the GUT gauge group through boundary conditions, the heavy vector superfields that have GUT-scale masses in 4D trinified theories instead have zero-modes with mass $M_c/2$ in the exact Higgsless limit. The $SU(3)_C \times SU(2)_W \times U(1)_Y$ quantum numbers of

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[2] Higher order combinations of the other spurions may generate a $(\mathbf{1},\mathbf{8},\mathbf{8})$; we assume a fine tuning of the sum of all such contributions.

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	(b_1, b_2, b_3)	$(\tilde{b_1}, \tilde{b_2}, \tilde{b_3})$
$(V, \Phi)_{321}$	(0,-6,-9)	(0,-4,-6)
$(V, \Phi)_{heavy}$	-	(-6,-2,0)
$H, \overline{H}$	$(\frac{3}{5},1,0)$	-
Matter	(6,6,6)	-
Total	$(\frac{33}{5},1,-3)$	(-6,-6,-6)

TABLE II: Contributions to the beta function coefficients from the zero modes (b_i) and the KK levels $(\tilde{b}_i)$ in our minimal scenario. Here Φ represents a chiral multiplet in the adjoint rep.

these states are given by

$$V_H \sim (\mathbf{1}, \mathbf{2}, 1/2) \oplus (\mathbf{1}, \mathbf{1}, 1) \oplus (\mathbf{1}, \mathbf{1}, 1) \oplus (\mathbf{1}, \mathbf{1}, 0) \oplus (\mathbf{1}, \mathbf{1}, 0) \ , \qquad (4.1)$$

where the hypercharges are shown here with their standard, rather than their GUT, normalization. KK modes of the ordinary MSSM vector superfields begin at M_c. The two towers of massive states are thus uniformly shifted with respect to each other by $M_c/2$. Each KK level in these towers consists of an $N = 2$ supersymmetric multiplet, which includes both a vector and a chiral superfield. The beta function contributions from these towers are indicated in Table II. Notice that the sum of all the KK gauge multiplet contributions to the beta functions is $(-6, -6, -6)$; if the two massive towers were degenerate level by level, they would affect gauge coupling running universally and have no effect on the quality of unification. However, the $M_c/2$ splitting separates these states into two subsets, each contributing nonuniverally to the beta functions. The shifted towers therefore provide a large number of threshold corrections to the differential gauge coupling running $\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu)$. There is no reason *a priori* to assume that these corrections will preserve gauge unification. In our trinified theory, we will see that they do.

While the individual α_i^{-1} experience power-law running above $M_c/2$, a remarkable feature of this tower of threshold corrections is that the $\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu)$ evolve logarithmically. This behavior was pointed out by Nomura, Smith and Weiner [15] in the context of a supersymmetric SU(5) GUT broken on a brane. Thus, theories of this type unify logarithmically, in contrast to the first examples of higher-dimensional gauge unification discussed in Refs. [16].
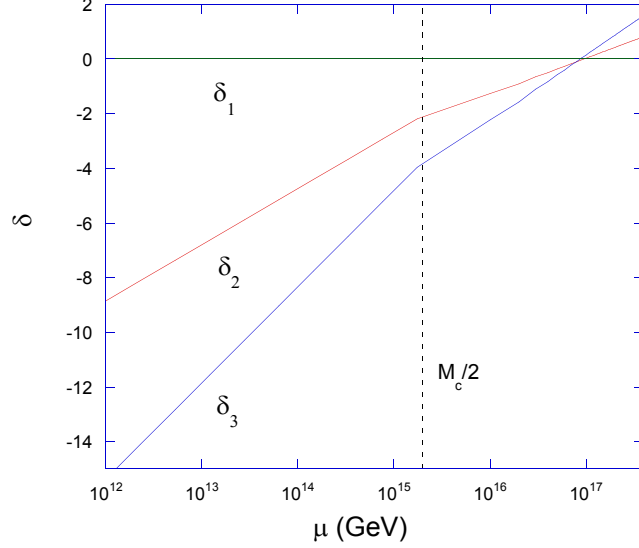
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FIG. 1: Gauge unification for $M_c = 4 \times 10^{15}$ GeV.

For our analysis, we follow the conventions of Ref. [15]: We first define gauge coupling differences with respect to α_1^{-1},

$$\delta_i(\mu) = \alpha_i^{-1}(\mu) - \alpha_1^{-1}(\mu). \qquad (4.2)$$

Unification occurs when $\delta_2 = \delta_3 = 0$. Above $M_c/2$, Eq.(4.2) can be written as

$$\delta_i(\mu) = \delta_i(M_c/2) - \frac{1}{2\pi} R_i(\mu) \qquad (4.3)$$

where $R_i(\mu)$ represents the differential logarithmic running between all the thresholds from $M_c/2$ up the the renormalization scale μ. For trinified gauge multiplets in the bulk only, we find

$$R_2(\mu) = -\frac{28}{5} \log(\frac{\mu}{M_c/2}) - 4 \sum_{0<nM_c<\mu} \log(\frac{\mu}{nM_c}) + 4 \sum_{0<(n+1/2)M_c<\mu} \log(\frac{\mu}{[n+1/2]M_c}), \qquad (4.4)$$

$$R_3(\mu) = -\frac{48}{5} \log(\frac{\mu}{M_c/2}) - 6 \sum_{0<nM_c<\mu} \log(\frac{\mu}{nM_c}) + 6 \sum_{0<(n+1/2)M_c<\mu} \log(\frac{\mu}{[n+1/2]M_c}). \qquad (4.5)$$

If the two towers of massive modes were degenerate, the last two terms in each of the equations above would have exactly canceled, and the R_i's would be the same as in the MSSM. The overall effect of the threshold corrections is to delay unification, as shown in Fig. 1.

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The shallower slopes above $M_c/2$ in Fig. 1 can be understood by rewriting Eq. (4.3) in the form.

$$\delta_i(\mu) = \delta_i(M_c/2) - \frac{1}{2\pi}\delta b_i \log(\frac{\mu}{M_c/2}) - \frac{1}{2\pi}\Delta\tilde{b}_{321} \sum_{nM_c/2<\mu} (-1)^n \log(\frac{\mu}{nM_c/2}) \qquad (4.6)$$

where we have used the fact that the difference in KK gauge multiplet beta functions $\Delta\tilde{b}_{321} = -\Delta\tilde{b}_{heavy}$. The first and second terms are negative and positive, respectively, and cancel in the MSSM at the unification point. The new term has positive coefficient $-\frac{1}{2\pi}\Delta\tilde{b}_{321}$. However, one may estimate the sum via integration, and one finds it is well approximated by $-(1 + \log(\mu/M_c))/2$. Thus, the new threshold corrections serve to reduce the effect of the second term (the MSSM differential logarithmic running) so that unification is delayed.

In the Higgsless limit, there are two significant physical scales in the theory: the compactification scale $1/R$, which determines the masses of the super-heavy states in the theory, and the 5D Planck scale, $M_*(5D)$, which determines where gravity becomes important. In Fig. 2, we show both the unification scale M_{GUT}, defined as the point at which $\alpha_1^{-1} = \alpha_2^{-1}$, and $M_*(5D)$, as a function of the compactification scale M_c. These scales are identical when $M_c \sim 2 \times 10^{15}$ GeV. For larger M_c, the 5D Planck scale is higher; in this case, one could introduce other, purely gravitational extra dimensions that again bring the higher-dimensional Planck scale in coincidence with M_{GUT}. For $M_c \lesssim 2 \times 10^{15}$ GeV, $M_*(5D)$ is lower than M_{GUT} and a field theoretic calculation of gauge coupling unification can no longer be trusted. For all values of M_c larger than 2×10^{15} GeV, the unification scale is increased relative to that of the 4D MSSM, *i.e.*, 2×10^{16} GeV. At its maximum value, 1.4×10^{17} GeV, the accuracy of gauge unification is $\sim 1\%$. This estimate assumes that brane-localized, higher-dimension kinetic energy operators have a negligible effect on the equality of the gauge couplings at the unification scale. Such an assumption is reasonable since these effects are volume suppressed by a factor of $\sim \pi M_*(5D)/M_c$ [15], which is generally large. Of course, the precise values of the operator coefficients are unknown, and one cannot rule out the possibility that such operators are simply not present in the theory.
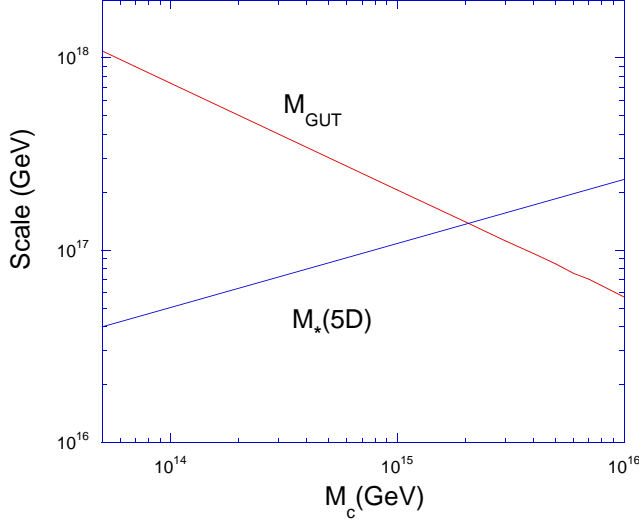
FIG. 2: Unification and 5D Planck scales as functions of M_c. For definitions of scales, see the text.

V. OTHER POSSIBILITIES

In the previous sections, we have allowed all exotic chiral superfields to be perfectly decoupled in the Higgsless limit. This was accomplished by restricting matter and Higgs multiplets to the $y = \pi R$ brane, and including the most general set of couplings to the symmetry breaking parameters. In this section, we discuss the alternative possibility that some (or all) of the **27**'s propagate in the bulk, along with the gauge multiplets. Assuming the same set of symmetry-breaking parameters on the πR brane, exotic fields now acquire masses of order the compactification scale, leaving the MSSM at low energies.

In general, a bulk matter field consists of an $N = 2$ hypermultiplet $\Psi = (\psi, \psi^c)$, where ψ and ψ^c are each left-handed, 4D $N = 1$ chiral superfields; in our case, these fields transform as a **27** and a $\overline{\mathbf{27}}$, respectively. We wish to argue that it is consistent within our framework to apply the following simple boundary conditions to elements of the **27** (and conjugate elements in the $\overline{\mathbf{27}}$) that we require to become massive:

$$\partial_5 \phi \, |_{y=0} = \phi^c \, |_{y=0} = \phi \, |_{y=\pi R} = \partial_5 \phi^c \, |_{y=\pi R} = 0 \ . \tag{5.1}$$

Here, ϕ and ϕ^c represented the scalar components of Ψ and Ψ^c, respectively. These boundary conditions are satisfied for

$$\phi = \sum_k N_k \cos(M_k y) \, \phi^{(k)}$$

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$$\phi^c = \sum_k N_k \sin(M_k y) \, \phi^{c(k)} \ , \tag{5.2}$$

where $N_k = (\pi R/2)^{-1/2}$, and $M_k = (k + 1/2) \, M_c$, for integer k. Of course, Eq. (5.2) solve the bulk equations of motion $\partial_M \partial^M \phi = 0$ provided that the KK modes satisfy the on-shell relation $p_k^2 = M_k^2$. Since supersymmetry is unbroken, the same conditions apply to the fermionic components as well.

To show that these boundary conditions are consistent, let us consider one possible ultraviolet completion. First, let us generalize our boundary conditions to

$$\partial_5 \phi \,|_{y=0} = 0 \quad , \quad \phi^c \,|_{y=0} = 0$$

$$(-\sin \eta \, \phi^c + \cos \eta \, \phi) \,|_{y=\pi R} = 0 \quad , \quad \partial_5(\cos \eta \, \phi^c + \sin \eta \, \phi) \,|_{y=\pi R} = 0 \tag{5.3}$$

which are satisfied by Eq. (5.2), if

$$\tan(M_k \pi R) = \cot \eta \ . \tag{5.4}$$

Notice that one linear combination of the fields in Eq. (5.3) satisfies Dirichlet boundary conditions at $y = \pi R$, while the orthogonal satisfies has Neumann boundary conditions. The precise linear combination is determined by the mixing angle η, which is a free parameter. Our desired boundary conditions are obtained from Eq. (5.3) in the limit that $\eta \to 0$.

Now consider the following 5D Lagrangian, with a brane-localized μ-term

$$\mathcal{L}_5 = \int d^4 \theta [\psi^\dagger \psi + \psi^{c\dagger} \psi^c] + \int d^2 \theta \, [\psi^c \partial_5 \psi + \frac{1}{2} \cot \eta \, \psi^2 \delta(y - \pi R)] \tag{5.5}$$

Here we have displayed the effective $N = 1$ supersymmetric Lagrangian, following the construction described in Ref. [17]. Extracting the purely scalar components, one finds

$$\begin{aligned} \mathcal{L}_5 = {} & F^\dagger F + \partial_\mu \phi^\dagger \partial^\mu \phi + F^{c\dagger} F^c + \partial_\mu \phi^{c\dagger} \partial^\mu \phi^c \\ & + \, [\, \phi^c \partial_5 F + F^c \partial_5 \phi + \cot \eta \, \phi F \, \delta(y - \pi R) + \text{ h.c.}] \end{aligned} \tag{5.6}$$

Aside from the bulk equations of motion for the auxiliary fields $F = \partial_5 \phi^{c\dagger}$ and $F^c = -\partial_5 \phi^\dagger$, one finds from the nonvanishing surface terms the boundary condition

$$-\phi^c \delta F|_{y=0} + (\phi^c + \cot \eta \, \phi) \delta F|_{y=\pi R} = 0 \tag{5.7}$$

which is clearly satisfied by the boundary conditions in Eq (5.3). Substituting out the auxiliary fields, one is left with the Lagrangian

$$\begin{aligned}
\mathcal{L} &= \partial_5 \phi^{c\dagger} \partial_5 \phi^c - \partial_5^2 \phi^{c\dagger} \phi^c + \phi_c^\dagger \partial_5^2 \phi_c + \partial_\mu \phi^{c\dagger} \partial^\mu \phi^c \\
&+ \partial_\mu \phi^\dagger \partial^\mu \phi - \partial_5 \phi^\dagger \partial_5 \phi - \cot\eta \, \delta(y - \pi R)(\partial_5 \phi^{c\dagger} \phi + \phi^\dagger \partial_5 \phi^c) \ .
\end{aligned} \tag{5.8}$$

Variation of the action with respect to ϕ leads to the further brane constraint

$$\partial_5 \phi^\dagger \delta\phi|_{y=0} + \partial_5(-\phi^\dagger + \cot\eta \, \phi^{c\dagger}) \, \delta\phi|_{y=\pi R} = 0 \tag{5.9}$$

which is satisfied by the remaining boundary conditions in Eq (5.3). Thus, our more general set of boundary conditions are consistent with this explicit brane Lagrangian. In particular, the simpler boundary conditions in Eq. (5.1) arise in the limit that the coupling $\cot\eta$ is allowed to become nonperturbatively large.

In the context of our previous discussion, the dimensionless brane coupling proportional to $\cot\theta$ arises at some order in the symmetry breaking spurions Φ. Generically,

$$W = -\frac{1}{2}\lambda(\Phi/\Lambda) \, \psi\psi \, \delta(y - \pi R) \ , \tag{5.10}$$

where, λ is a dimensionless coupling, and $\cot\eta$ is identified with $\lambda\Phi/\Lambda$. Any exotic field that decoupled in our earlier construction, will receive a brane coupling proportional to $\cot\eta$ in the present one. Thus, in the $\eta \to 0$ limit, we recover the boundary conditions of Eq. (5.1) applied to that particular field, whose zero mode obtains a mass of $M_c/2$.

If we take this completion literally, then we would want to restrict $\cot\eta$ by the condition that the coupling λ remain perturbative. However, we are not wedding ourselves to any particular origin for the boundary conditions. We will take the example just discussed as motivation for the consistency of Eq. (5.1), and work in the exact $\eta = 0$ limit. The reader who disagrees with this approach may simply consider our results an approximation to the explicit ultraviolet completion discussed above when $\cot\eta$ is taken to be somewhat strongly coupled.

In the case where the bulk $\mathbf{27}$'s are the three standard model generations, the exotic N, E and B fields will become massive given our choice of brane spurions. Our results for gauge unification will not be affected since these fields form the complete SU(5) reps $\mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{1} \oplus \mathbf{1}$. Another possibility is to place the $\mathbf{27}$ and $\overline{\mathbf{27}}$ Higgs multiplets in the bulk. In

this case, we have a tower of KK modes beginning at $M_C/2$ for the massive components, and a tower beginning at M_C for those components with massless zero modes. This leads to an additional threshold correction of the type discussed in Section IV. We find that this tends to spoil unification for values of $1/R$ that are significantly smaller than the conventional supersymmetric unification scale, $M_u = 2 \times 10^{16}$ GeV. Thus, this possibility may be realized if $1/R$ and M_u are within a factor of a few of each other so that unification is preserved to good approximation.

VI.   CONCLUSIONS

The breaking of gauge symmetries through the choice of consistent boundary conditions on an extra dimensional interval provides a powerful new tool for model building. Unlike the orbifold case, a more general choice of boundary conditions allows one to reduce the rank of the bulk gauge group. Aside from the breaking of electroweak symmetry [2, 3], this approach is naturally of interest in the breaking of grand unified and other gauge extensions of the standard model that have gauge groups with rank greater than four. We have demonstrated this explicitly in the case of gauge trinification. We obtained boundary conditions necessary to break the trinified gauge group directly down to that of the standard model, while preserving the GUT-scale relation $\sin^2 \theta_W = 3/8$. Symmetry breaking was introduced consistently in terms of spurions localized on the πR brane. In the Higgsless limit, in which these spurions are taken to infinity, the massive gauge multiplets have zero-modes at $M_c/2$, where M_c is the compactification scale. In the same limit, all exotic matter and Higgs fields are decoupled from the theory, and Higgs-mediated proton decay is avoided. We retain the light Higgs doublets of the MSSM, so that light fermion masses may be easily obtained. By placing the gauge multiplets in the bulk, there is power law running due to the KK modes. As in other 5D unified theories with gauge symmetries broken on a boundary [15], we find that the running of the differences $\alpha_i^{-1} - \alpha_j^{-1}$ remains logarithmic. For the massive gauge fields in our trinified theory, we find that unification is preserved, and that the scale at which the couplings unify is increased. For $M_c \sim 2 \times 10^{15}$ GeV, the gauge couplings unify at the 5D Planck mass 1.4×10^{17} GeV, with a percent accuracy at the one-loop level.

Acknowledgments

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